Received: *May 28, 2001*, Accepted: *May 31, 2001*

String theory and quintessence

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Abstract: We discuss the obstacles for defining a set of observable quantities analogous to an S-matrix which are needed to formulate string theory in an accelerating universe. We show that the quintessence models with the equations of state $-1 < w < -1/3$ have future horizons and may be no better suited to an S-matrix or S-vector description. We also show that in a class of theories with a stable supersymmetric vacuum, a system cannot relax into a zero-energy supersymmetric vacuum while accelerating if the evolution is dominated by a single scalar field with a stable potential. Thus describing an eternally accelerating universe may be a challenge for string theory as presently defined.

Keywords: M-Theory, Superstring Vacua, Cosmology of Theories beyond the SM.

Contents

1. Introduction

The development of string theory over the last three decades has convincingly demonstrated the existence of a precise mathematical structure that includes low energy gravity, black holes, and a wide variety of interesting structures. Furthermore it is a quantum theory and therefore demonstrates the consistency of gravitation and quantum mechanics. That has been especially interesting in the context of black hole physics where such concepts as black hole complementarity and the holographic principle can be subjected to rigorous tests.

When it comes to the foundations of cosmology string theory has not proved as fruitful. The theory as presently formulated is not background independent. Each background seems to have its own idiosyncratic description as illustrated by the matrix theory description of flat 11-dimensional M-theory; the various toroidal compactifications which make use of higher dimensional gauge theories; the CFT description of AdS space and little string theories for certain linear dilaton backgrounds.

Thus far we have failed to include interesting cosmological backgrounds within the framework of the controllable mathematics of string theory [1]. In particular no deSitter space backgrounds have been found which could serve as models of the currently accelerating universe [2]. In this paper we will discuss a possible reason why accelerating backgrounds have not been found. As we will see the same reasons would preclude eternal quintessence-like behavior.

The message that we wish to convey is not that deSitter space or quintessence is impossible but rather that the current mathematical framework may not be the right one for cosmology. The view that deSitter space may require fundamental revisions of our ideas about string theory has also been forcefully expressed by Banks [3]. We will argue that this is part of a pattern that effects not only those theories with positive cosmological constants but also any geometry with a future horizon. As we will see this includes quintessence-like geometries.

2. The observables of string theory

Just as the degrees of freedom of string theory are idiosyncratic to each background, so are the "observables" that string theory allows us to compute. In asymptotically flat space-time string theory is a prescription for calculating S-matrix elements relating asymptotically free particle states. As far as we know, these S-matrix elements are the only rigorously defined quantities of the theory. The S-matrix data consists of a list of all the stable objects in the theory and the transition amplitudes between asymptotic states of these objects. One essential requirement for the existence of an S-matrix is to have an asymptotically large space at infinity in which degrees of freedom can separate into a collection of non-interacting particles.

AdS space is another example in which observables of string theory have been identified, namely the boundary correlators of bulk fields. These boundary correlators are very similar to S-matrix elements and again rely on an infinite asymptotic space in which the bulk degrees of freedom can separate into free particles.

There is some evidence that AdS string theory may allow a larger class of well defined quantities. The AdS/CFT correspondence relates the correlators of local gauge invariant CFT fields to the boundary correlators in AdS. But there are other quantities on the CFT side which appear to be well defined. These quantities are gauge invariant but non-local and are typified by closed Wilson loops. According to [4] these finite size non-local objects in the CFT are dual to bulk degrees of freedom far from the boundary of AdS. However we understand very little about the Holographic dictionary relating non-local CFT operators to small objects in the AdS interior.

Neither asymptotically flat space-time nor AdS space provides a realistic framework for cosmology. Thus let us turn to the more relevant example of familiar FRW cosmology. It is quite evident that closed FRW spaces which begin and end with bangs and crunches do not permit asymptotically free particles. Thus, with the present concepts available in string theory we would not know how to formulate the theory in such a background.

Flat-space FRW is not much better in the initial state but the final states are describable in terms of asymptotically free particles. A possible framework for cosmology has been suggested by Witten [5]. The idea is to assume that the initial state of the universe is unique, $|U\rangle$. The final state consisting of free asymptotically well separated particles would be described in terms of a Fock space of asymptotic out-fields. The S-matrix would then be replaced by an S-vector describing the final state amplitudes. In this case correlation functions of (almost) asymptotic fields could be

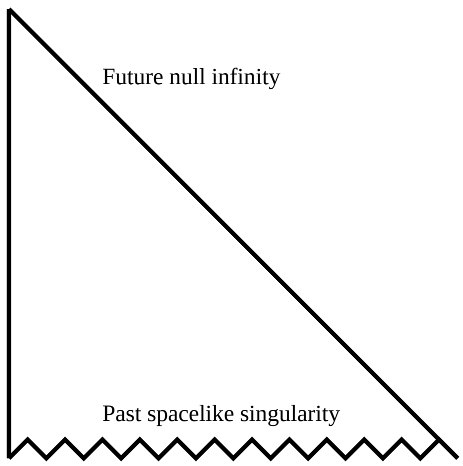

Figure 1: Causal structure of FRW spacetime.

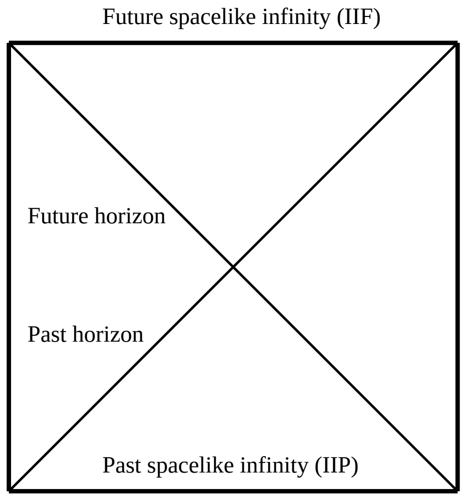

Figure 2: Causal structure of deSitter spacetime.

measured by observers stationed at different points. Furthermore in flat FRW space, every set of points has at least one point in the causal future of the set. In other words it is possible to send signals from every point in the set to a common point where the data is collected and correlated. Thus the S-vector would have operational meaning. For purposes of comparison with other cosmologies we illustrate the causal structure of flat FRW in the Penrose diagram of figure 1.

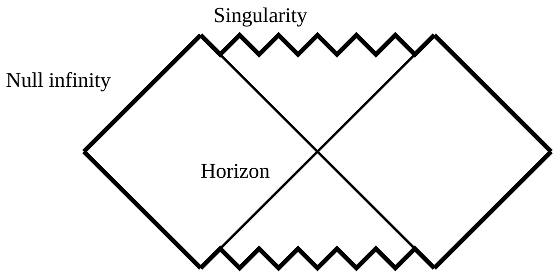

Figure 3: Causal structure of Schwarzschild spacetime.

Now let us turn to deSitter space. The structure of deSitter space can be described by its Penrose diagram shown in figure 2. For comparison we also show the Penrose diagram of a Schwarzschild black hole in figure 3. In the black hole case there is an infinite asymptotically flat region outside the black hole but in the deSitter space case there is not. Another difference is that the black hole has a space-like singularity while the deSitter space geometry has a space-like causal boundary which represents the infinite inflated future (IIF). The IIF and the future black hole singularity are causally very similar. In particular consider two events or measurements at points a and b in each geometry. If the events are too close to the IIF or future singularity then the causal futures of these two points do not have any common points. In other words there is no possibility of collecting data from the two measurements. An implication of this is that correlations between quantities at a and at b are unmeasurable by any real observer. For this reason an S-matrix connecting the IIF to the IIP (infinitely inflated past) or to a unique big bang has no observable meaning.

The correlations between a, b could nevertheless have formal meaning. According to one view we can foliate each space by space-like slices and formulate quantum field theory as a hamiltonian theory with state vectors being defined on each slice. If a and b are relatively space-like, the quantum fields at the two points commute and in a formal sense can be simultaneously measured. Maldacena and others have suggested that boundary correlators defined in terms of fields in the IIP and the IIF may exist and be described by some kind of space-like holography.

This seems unlikely to us. It is well known and generally accepted by now, that in the context of black holes this reasoning is very dangerous and either leads to information loss for observers outside the black hole or information duplication for observers who fall through the horizon. Either of these is a violation of the laws of nature. The principle of Black Hole Complementarity states that the degrees of freedom behind and in front of the horizon can not be independent but must be the same objects seen in different gauges. The same points have been emphasized in [3]. We believe similar kinds of arguments can be made about any future horizons so that independent degrees of freedom on space-like slices close to the "end of time" do not make sense.

There is another way to think about quantum theory in deSitter space which is suggested by the black hole complementarity principle. In the black hole case we give up trying to simultaneously describe both sides of the horizon but instead look for a self contained description of the exterior of the black hole. For purposes of an external observer the world ends on a hot membrane called the horizon. More exactly, black holes are merely unstable resonances in scattering amplitudes and information is never lost behind the horizon. In a similar spirit one can define a causal diamond of an observer in deSitter space as shown in figure 3. The physics in the causal diamond is described in terms of a static metric describing a static sealed cavity with metric

$$ds^2 = -\left(1 - \frac{\rho^2}{R^2}\right) d\vartheta^2 + \frac{d\rho^2}{1 - \rho^2/R^2} + \rho^2 d\Omega_2 \,. \tag{2.1}$$

The cavity consists of the region $r < R$ and the boundary at $r = R$ is a conventional Rindler-like horizon. The cavity has a finite temperature which has the formal value

$$T = \frac{1}{2\pi R} \,. \tag{2.2}$$

The actual proper temperature experienced by an observer at r varies from $T = 1/2\pi R$ at $r = 0$ to infinity at $r = R$. Near the horizon the proper temperature is given by

$$T_{\text{proper}} \sim \frac{1}{2\pi D} \,, \tag{2.3}$$

where D is the proper distance to the horizon. It is obvious that in a thermal cavity of finite size, an S-matrix can not be defined.

JHEP06(2001)003

It is possible that there is a holographic description of the cavity and that there are quantities that describe the interior of the cavity in much the same way that the closed Wilson loops describe the interior of AdS. However we know of no such formulation.

There are other possibly related reasons for suspecting that string theory as we know it can not be defined in deSitter space. All definitions of string theory involve taking the limit of infinitely many degrees of freedom. For example in its original form, string theory is defined in terms of 2-dimensional conformal field theories. These theories can be thought of as the limiting behavior of regulated world sheet theories with finitely many degrees of freedom. The conformal limit is the limit of infinitely many degrees of freedom.

Most quantities will not have limits as the regulator is removed. What we have learned is that the observables of string theory are just those quantities, namely on-shell S-matrix elements, which do have limits. Even these quantities will only have limits if the background target space geometry is suitably chosen so as to flow to a non-trivial fixed point. deSitter space is not such a fixed point.

Non-perturbative definitions of string theory in terms of matrix theory or the AdS/CFT correspondence also require the number of degrees of freedom to tend to infinity. As in perturbative string theory the only objects which we expect to have limits are S-matrix elements.

On the other hand Banks [3] has emphasized that the finite entropy of deSitter space means that the universe has only a finite number of degrees of freedom (see also [6]). This is probably related to the lack of an asymptotic space in the thermal cavity. In any case String Theory as we know it is not defined in deSitter space. One other indication that string theory and deSitter space are not easily combined was given by Maldacena and Nunez [7].

Assuming that these arguments can be put on a firm footing and that observation continues to point to a non-vanishing positive cosmological constant a serious crisis between theory and observation may materialize.

3. Q-space

The theory of quintessence [8, 9] has been put forward as an alternative to a positive cosmological constant. According to this theory the dark energy of the universe is dominated by the potential of a scalar field ϕ which is still rolling to its minimum at $V = 0$. Typically the minimum is at $\phi = \infty$ and the scalar potential may have a form such as

$$V(\phi) \sim \exp(-c\sqrt{\alpha'}\phi)\,, \tag{3.1}$$

where ϕ is a canonically normalized scalar field and c is a numerical constant of order unity. The theory can also be parameterized by an equation of state of the usual form

$$P = w\epsilon\,, \tag{3.2}$$

where P and ϵ denote pressure and energy density. Recall that a cosmological constant corresponds to $w = -1$, matter domination to $w = 0$ and radiation dominance to $w = 1/3$. Quintessence gives rise to equations of state with

$$-1 < w < -\frac{1}{3}\,. \tag{3.3}$$

The observational evidence for a cosmological constant is really a bound on w:

$$-1 < w_{\rm observed} \lesssim -\frac{2}{3}\,. \tag{3.4}$$

Thus we are not yet at the point where we need to postulate a genuine cosmological constant.

The purpose of this paper is to analyze the causal structure of universes with w in the allowable range and remaining so in the remote future. What we will find is that universes of this type have future horizons and are no better suited to an S-matrix or S-vector description than is deSitter space. For simplicity we will work in 4 spacetime dimensions. Our results however extend straightforwardly to other dimensions.

Isotropic and homogeneous universes are described by the FRW metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2\right)\,. \tag{3.5}$$

Here $k = 0, \pm 1$ is the spatial curvature of the universe, $d\Omega_2$ the line element on a unit 2-sphere S^2 and $a(t)$ the time-dependent scale factor, which measures the proper size across the celestial sphere. We choose the units such that a has dimensions of length. The gravity-matter field equations then reduce to

$$\begin{aligned} 3H^2 + 3\frac{k}{a^2} &= \frac{8\pi}{M_4^2}\epsilon \\ \dot\epsilon + 3H(\epsilon + P) &= 0\,, \end{aligned} \tag{3.6}$$

where the Hubble parameter $H = \dot a/a$ is the logarithmic time derivative of the scale factor, and $M_4 \sim 10^{19} GeV$ is the Planck scale. The equation of state $P/\epsilon = w$ allows to solve the second of eq. (3.6), yielding

$$\epsilon = \epsilon_0 \left(\frac{a_0}{a}\right)^{3(1+w)}\,. \tag{3.7}$$

The gauge choice $k = 0, \pm 1$ implies that $\epsilon_0 \sim M_4^{-4}$ to ensure the validity of (3.6) at subplanckian scales. Here $a_0 \geq M_4^{-1}$ is an integration constant parameterizing the initial size of the universe. With this parameterization, (3.6) are valid for times $t \geq M_4^{-1}$.

JHEP06(2001)003

The second time derivative of the scale factor a is

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3M_4^2}(1+3w)\epsilon\,, \tag{3.8}$$

so that for $w > -1/3$ the expansion of the universe decelerates, for $-1 < w < -1/3$ it accelerates and for $w = -1/3$ it is inertial. One can now see that observers in all accelerated models must have future horizons. For any two objects separated by a fixed comoving distance r in a universe whose expansion is accelerating, their relative proper speed will reach the speed of light after some time, and they will cease to communicate. This cessation of communication cannot occur in models where expansion decelerates, where in fact the communication becomes less relativistic as time goes by. The borderline case $w = -1/3$ describes constant expansion rate, where the comoving observers move with constant speed relative to one another. Their communication is possible, but it may be difficult to maintain forever because they recede away from each other.

We now explicitly construct the spacetime geometry for $-1 < w \leq -1/3$ models. For simplicity, we begin with the spatially flat cases ($k = 0$), and note that the conclusions are qualitatively the same for the spatially open ($k = -1$) cases as well. The spatially closed cases ($k = 1$) will be treated separately.

When $k = 0$ the eqs. (3.6) and (3.7) yield the scale factor

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{2/[3(1+w)]}\,, \tag{3.9}$$

where $t_0 \geq M_4^{-1}$ is an integration constant. Because $1 + w > 0$, all of the solutions (3.9) have a curvature singularity at $t = 0$ where the scalar curvature $R = 6\dot{H} + 12H^2$ diverges. The scale factor is unbounded as $t \to \infty$. The curvature goes to zero and locally the flat space approximation becomes progressively better. The decelerating FRW models satisfy $w > -1/3$ and have a spacelike past singularity. Their future infinity does not have a spacelike portion, and so the observers never have any future horizons, but can see arbitrarily far away if they wait long enough (see figure 1).

The situation is rather the opposite in universes whose expansion accelerates under the influence of a quintessence-like stress energy $-1 < w < -1/3$. To show this we now construct the Penrose diagram describing such spacetimes. With (3.9), the $k = 0$ metric (3.5) is

$$ds^2 = -dt^2 + a_0^2 \left(\frac{t}{t_0}\right)^{4/[3(1+w)]} \left(dr^2 + r^2 d\Omega_2\right). \tag{3.10}$$

The standard tool for determining the causal structure of a homogeneous and isotropic universe is to conformally map it on a part of the Einstein static universe [10]. The Einstein static universe is a direct product of a spatial sphere S^3 of constant radius and an infinite time axis, with the metric $ds^2 = -d\tau^2 + d\chi^2 + \sin^2(\chi)d\Omega_2$. Therefore its causal structure is that of an infinite cylinder $R \times S^3$. The part of it which describes the causal structure of some arbitrary homogeneous and isotropic universe is bounded by the images of the singularities and/or past and future causal boundaries. Since (3.10) has a past singularity and becomes locally flat as $t \to \infty$, its boundary consists of precisely these regions.

For clarity's sake, we will determine the causal structure of (3.10) by finding the conformal map to the Einstein static universe in two steps. First we go to the conformally flat metric $ds^2 = \omega^2(\bar{x})\eta_{\mu\nu}d\bar{x}^\mu d\bar{x}^\nu$, and relate the flat metric to the Einstein static one. The first step is

$$(1+3w)\bar{t} = 3(1+w)\left(\frac{t}{l}\right)^{(1+3w)/[3(1+w)]}$$
$$\omega(\bar{t}) = l\left(\frac{1+3w}{3(1+w)}\bar{t}\right)^{2/(1+3w)} . \tag{3.11}$$

Here $\bar{t}$ is dimensionless, and since $-1 < w < -1/3$, it is negative and inversely proportional to t, running however from $-\infty$ to 0 as t runs from 0 to ∞. Therefore, the $\bar{t}$-axis has the same orientation as the t-axis. Here $l = \left(a_0 M_4^{2/[3(1+w)]}\right)^{[3(1+w)]/(1+3w)}$ has dimension of length.

The second step is defined by

$$r = \frac{1}{2}\left(\tan\left(\frac{\chi+\tau}{2}\right) + \tan\left(\frac{\chi-\tau}{2}\right)\right)$$
$$\bar{t} = \frac{1}{2}\left(\tan\left(\frac{\chi+\tau}{2}\right) - \tan\left(\frac{\chi-\tau}{2}\right)\right) . \tag{3.12}$$

Since $r \in [0,\infty)$, $\bar{t} \in (-\infty, 0)$, and $\chi \in [0,\pi]$, it is straightforward to verify that $\tau \in [-\pi, 0]$. We can now put together these formulas and write

$$ds^2 = l^2\left(\frac{6(1+w)}{|1+3w|}\right)^{4/|1+3w|} \frac{[\cos(\frac{\chi-\tau}{2})\cos(\frac{\chi+\tau}{2})]^{4/|1+3w|-2}}{4\ \sin^{4/|1+3w|}(|\tau|)}\left(-d\tau^2 + d\chi^2 + \sin^2(\chi)d\Omega_2\right) \tag{3.13}$$

for the metric, and

$$\frac{|1+3w|}{6(1+w)}\left(\frac{t}{l}\right)^{|1+3w|/[3(1+w)]} = \left(\tan(\frac{\chi-\tau}{2}) - \tan(\frac{\chi+\tau}{2})\right)^{-1} \tag{3.14}$$

for the relationship between the comoving time t and the Einstein static universe time τ. The causal structure of (3.10) is a portion of $R \times S^3$ bounded by the future and the singular regions of (3.10). The eq. (3.14) together with the first of eq. (3.12) is the key for determining this boundary, in particular the power -1 on the right-hand side. Using this, the future limit $t \to \infty$ for any given r maps on the line

$\tan(\frac{\chi-\tau}{2}) = \tan(\frac{\chi+\tau}{2})$. This is the latitude circle $\tau = 0$ on the cylinder. Because the spacetime (3.10) ends there, we must discard the portion of the cylinder above it. Next, the singularity resides in the limit $t \to 0$ for any fixed r. By (3.12), (3.14) it maps onto the line $\tan(\frac{\chi-\tau}{2}) \to \infty$, or therefore $\tau = \chi - \pi$. This is the null semi-circle connecting the points $(-\pi, 0)$ and $(0, \pi)$ on the cylinder, and since it is the past we must throw out the portion of the cylinder beneath it. Unwrapping the remainder, we get the causal structure in figure 4.

We have suppressed the angular dependence in figure 4, meaning that each point on it corresponds to an angular S^2. It is evident from this figure that all the universes described by (3.10) descend from a past null singularity, and evolve towards a future spacelike infinity. Therefore, any observer must have a future horizon: it is precisely the null line starting in the upper left corner of the diagram and flowing towards the singularity, and corresponds to the portion of the circle $\tau = \pi - \chi$ above the singular circle $\tau = \chi - \pi$. An observer would find the universe at any given finite time to be of finite size - she could only causally explore the interior of the diamond bounded by the horizon and the singularity. However, unlike in deSitter space, she would not lack elbow room in her box. The proper size of the horizon is not constant but grows in time,

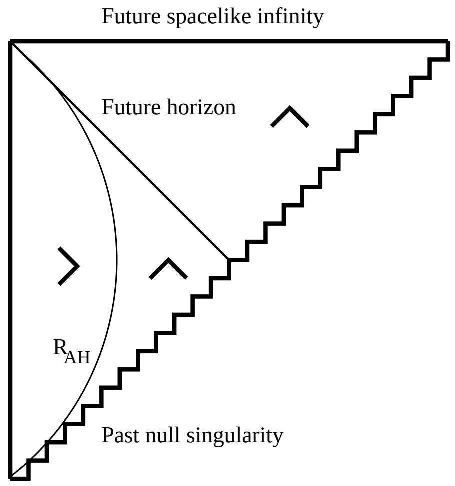

Figure 4: Causal structure of a spatially flat universe dominated by $-1 < w < -1/3$ matter.

$$L_H = a(t) \int_t^\infty \frac{dt'}{a(t')} = \frac{3(1+w)}{|1+3w|} t\,, \tag{3.15}$$

which shows that the volume of any spacelike hypersurface inside the causal diamond grows extremely large with time! In other words, the proper volume of the region of space near the upper left corner of the diagram in figure 4 is tremendously large, and since the horizon is moving away (3.15), it is getting even larger. This is easy to understand intuitively: even though the expansion of the universe (3.10) is accelerating, the acceleration rate decreases with time as $\ddot{a}/a \sim 1/t^2$. Thus at a later time it takes longer for the cosmic acceleration to increase the proper speed between comoving observers to the speed of light.

A proposal for a holographic description of cosmology has been made in [11], and generalized to arbitrary spacetimes in [12]. An important tool for defining holographic screens is the apparent horizon. We determine its location in these spacetimes. It is the hypersurface where at least one family of null lines has vanishing expansion. To find it for the solution (3.10), consider a sphere of radius ar with area $A \sim a^2(t)r^2$, and its variation along the null radial lines $dt = \pm a(t)dr$, with the signs determining the orientation of the lines. The gradient of A along a light ray is $A' \sim a'r + ar'$ where the prime denotes the derivative with respect to the affine parameter of the null line. Using these equations,[1] the comoving size of the apparent horizon is $r = \dot{a}^{-1}(t)$, and so the proper apparent horizon size is

$$\mathcal{R}_{AH} = \frac{1}{H} = \frac{3(1+w)}{2}t\,. \tag{3.16}$$

Dividing by L_H (3.15), we get $\mathcal{R}_{AH}/L_H = |1+3w|/2 < 1$ for $-1 < w < -1/3$, and so the apparent horizon is always inside the future horizon. On the diagram of figure 4, it is represented by the arc $\mathcal{R}_{AH}$ between the lower left corner and the upper left corner. The interior of the apparent horizon is a normal region, and its exterior, including parts of the causal diamond, is an anti-trapped region. Following Bousso [12], we denote this by the $>$ symbols, where the legs point in the direction of decreasing geodesic expansion θ of the lightrays. As we have said above, all of these conclusions directly apply for the spatially open cases $k = -1$.

The main difference between the models with $-1 < w < -1/3$ and the cosmological term $w = -1$ is in the presence of the null singularity in the past. For $w = -1$, that null surface is regular, and can be extended across since it is just a past horizon. Instead, the solutions with $-1 < w < -1/3$ all end there, and so are past inextendable, fully describing the geometry. We can also see how these solutions differ from the decelerated FRW universes. For these $1 + 3w > 0$ implies the absence of the power -1 on the RHS of the analogue of eq. (3.14). This turns figure 4 upside-down, and yields a past spacelike singularity and a future null infinity as its boundaries.

Now we turn to the borderline case $w = -1/3$. It is a hybrid of the previous two cases, in that its past is quintessential, with a past null singularity, and its future is similar to a usual FRW universe with a future null infinity. The scale factor metric for this case is $a(t) = a_0 t/t_0$, and so the analogue of eq. (3.11) is

$$\begin{aligned} \bar{t} &= \frac{t_0}{a_0}\ln\left(\frac{t}{t_0}\right) \\ \omega(\bar{t}) &= a_0 \exp\left(\frac{a_0 \bar{t}}{t_0}\right) \end{aligned} \tag{3.17}$$

Combining this and (3.12), we find

$$\ln\left(\frac{t}{t_0}\right) = \frac{a_0}{2t_0}\left(\tan\left(\frac{\chi+\tau}{2}\right) - \tan\left(\frac{\chi-\tau}{2}\right)\right). \tag{3.18}$$

The range of the angular variables is the same as before, $\chi \in [0, \pi]$, but now $\tau \in (-\pi, \pi)$ because $\ln(t/t_0)$ can be both positive and negative. It is straightforward to

[1] An alternative way to determine $\mathcal{R}_{AH}$ is to find the hypersurface $A = \text{const}$ whose normal is null. Then its area is independent of the null coordinate along the normal, and its gradient vanishes, implying that this hypersurface is the apparent horizon.

check that the singularity maps on the past null semi-circle $\tau = \chi - \pi$, and the future infinity maps on the future null semi-circle $\tau = \pi - \chi$. The boundary is completed by the intersection of these two curves at $\chi = \pi, \tau = 0$, which corresponds to spatial infinity. The spacetime is the interior of the boundary, and when we unwrap it we find the structure depicted in figure 5.

Since the spacetime has a past null singularity a future null infinity no comoving observer living in it will have a horizon. The space is infinite, but "just so" since even an infinitesimal acceleration would raise the horizon, confining any observer into her box.

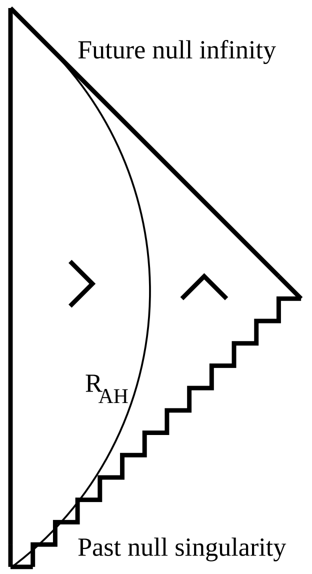

Figure 5: Causal structure of a spatially flat universe dominated by $w = -1/3$ matter.

Although in this case there is no event horizon, there is an apparent one. Since $a \sim t$, the radius of the apparent horizon is

$$\mathcal{R}_{AH} = t\,. \tag{3.19}$$

In the comoving coordinates, this corresponds to $r_{AH} = 1$, and is again the arc between the lower left corner and the upper left corner, denoted $\mathcal{R}_{AH}$ in figure 5. Its interior is a normal region, and its exterior is an anti-trapped region.

The situation is essentially the same for the open universe case $k = -1$. There the form of the metric

$$ds^2 = -dt^2 + a_0^2 \left(\frac{t}{t_0}\right)^2 \left(\frac{dr^2}{1+r^2} + r^2 d\Omega_2\right) \tag{3.20}$$

might for a moment deceive one into thinking that the solution is just a Milne wedge of the flat Minkowski space in an accelerated reference frame. However, this is not so because when one tries to extend the metric (3.20) across the Milne boundary, one finds a deficit angle on the S^2, which translates into a curvature singularity there. Hence, this solution too has a null singularity in the past, and its causal structure is also depicted by figure 5.

We now consider the spatially closed cases with $k = +1$ in (3.5). When $-1 < w < -1/3$ combining eqs. (3.6) and (3.7) we get the master equation governing the geometry of these solutions, which is

$$\dot{a}^2 + 1 = \frac{8\pi\epsilon_0 a_0^2}{3M_4^2} \left(\frac{a}{a_0}\right)^{|1+3w|} . \tag{3.21}$$

Thus $a(t) > 0$ throughout the evolution of the universe, implying that all of these solutions are completely nonsingular. Further, the evolution is completely symmetric under $t \to -t$, and for very large values of a (in the far past and the far future) the spatial curvature term is negligible. Thus in these limits the future of these geometries asymptotes that of (3.10), and by time reversal symmetry, so does the past. Hence the causal structure corresponds to a finite segment of the Einstein static cylinder, bounded by two latitutude circles $\tau = \pm\tau_0$, and the universe has both a past and a future spacelike infinity. As a result, any observer would see horizons, whose structure is more complicated than in the spatially flat case. Thus her life would again be confined to a box, whose size however varies with time. For $t < 0$, the box would be shrinking down from an infinite size in the past, to rebound at $t = 0$ and begin to expand out again. This can be seen explicitly in an example where $w = -2/3$, which would correspond to a universe eternally dominated by a frustrated network of domain walls. The solution is

$$a(t) = \frac{3M_4^2}{8\pi\epsilon_0 a_0} + \frac{2\pi\epsilon_0 a_0}{3M_4^2} t^2 . \tag{3.22}$$

It has a bounce at $t = 0$, where it reaches the minimum size $a_{\rm min} = \frac{3M_4^2}{8\pi\epsilon_0 a_0}$ and rapidly approaches the $w = -2/3$ flat solution (3.10) when $t \to \pm\infty$. The boundary structure of these models is therefore similar to that of deSitter space, figure 2.

In the special limit $w = -1/3$, the solutions degenerate into two interesting cases. If $\frac{8\pi\epsilon_0 a_0^2}{3M_4^2} = 1$, the geometry is that of the whole Einstein static universe with $\dot a = \ddot a = 0$. This geometry does not have any horizons, because the space has finite size, and time runs forever, so any observer would eventually see all of the space. If $\frac{8\pi\epsilon_0 a_0^2}{3M_4^2} > 1$, we get the other class of solutions where the scale factor undergoes either linear expansion or linear contraction, depending on the sign of t, $a(t) = \sqrt{\frac{8\pi\epsilon_0 a_0^2}{3M_4^2} - 1}\; t = \hat a_0 t/t_0$. The conformal map to the Einstein static universe in this case is simple, involving the single step $\tau = (\hat a_0/t_0)^{-1}\ln(t/t_0)$. The singularity $t = 0$ then maps onto a latitude circle at $\tau = \pm\infty$, implying that the causal structure is given by the infinite cylinder $R \times S^3$, but with a spacelike singularity either in the infinite past or future.

Given that the observer inhabiting the universes with $-1 < w < -1/3$ lives inside a spacetime box, we may ask what are the proper coordinates which chart out only the interior of this box. After all, since all of the observer's physical reality is confined inside the box, that is all she cares about. A simple way to find such coordinates for the spatially flat cases (3.10) is to use their conformal relation to the spatially flat slicing of deSitter space, and then go to the static coordinates describing the causal diamond in deSitter space, since it is a subset of the spatially flat deSitter chart. The appropriate maps are

$$\begin{aligned} t &= \frac{|1+3w|}{3(1+w)} R\Big(1 - \frac{\rho^2}{R^2}\Big)^{3(1+w)/[2|1+3w|]} e^{3(1+w)\vartheta/[|1+3w|R]} \\ r &= \frac{\rho}{R}\Big(1 - \frac{\rho^2}{R^2}\Big)^{-1/2} e^{-\vartheta/R} , \end{aligned} \tag{3.23}$$

where $R = \left(\frac{|1+3w|}{3(1+w)}\right)^{2/|1+3w|} l$. The metric inside the box $\rho \le R$ is

$$ds^2 = \left(1 - \frac{\rho^2}{R^2}\right)^{3(1+w)/|1+3w|} e^{6(1+w)\vartheta/[|1+3w|R]} \left(- \left(1 - \frac{\rho^2}{R^2}\right) d\vartheta^2 + \frac{d\rho^2}{1 - \frac{\rho^2}{R^2}} + \rho^2 d\Omega_2 \right) . \tag{3.24}$$

The volume of the box grows unbounded as $\vartheta \to \infty$ - indeed, the area of a sphere of constant radius R increases linearly with proper time. The "effective" curvature emerges from the scale set by the initial size of the universe l. Close to the horizon, the conformal factor in (3.24) approaches $(L_H/R)^2$, as seen from (3.15 and (3.23). This suggests that at late times the description of the short-distance physics near the horizon of (3.24), with characteristic scale $\ll L_H$, is similar to deSitter space. In this case time variation of L_H can be neglected. For short distance phenomena it merely sets the overall scale of the geometry.

To make this more precise, we use the limit when the equation of state approaches that of cosmological constant, $w = -1 + 2\epsilon/3$. Then the solution approaches deSitter metric,

$$ds^2 = \left(1 - \frac{\rho^2}{R^2}\right)^{\epsilon} e^{2\epsilon\vartheta/R} \Big(- \left(1 - \frac{\rho^2}{R^2}\right) d\vartheta^2 + \frac{d\rho^2}{1 - \rho^2/R^2} + \rho^2 d\Omega_2 \Big) . \tag{3.25}$$

The variation of the conformal factor anywhere away from singularity is slow for this case. In other words, this limit expands out the geometry near the horizon of (3.24) towards the interior of the causal diamond. Indeed, from (3.9) we see that $a = a_0 (t/t_0)^{1/\epsilon}$, and when $\epsilon \to 0$ the correct way to take the limit is to rescale $a_0 \to a_0 \epsilon^{1/\epsilon}$ and shift time $t \to t + t_0/\epsilon$. Then, $a \to a_0 (1 + \epsilon t/t_0)^{1/\epsilon} \to a_0 \exp(t/t_0)$, and from (3.15), $L_H = \epsilon t \to t_0 + \epsilon t$, so that the horizon is nearly static. Hence deSitter approximation becomes good for most of its interior, and it behaves like an almost static cavity with hot walls, whose temperature is $T \sim 1/2\pi(t_0 + \epsilon t)$. The walls are moving away with a constant infinitesimal speed ϵ.

4. Supersymmetry versus Q-space

We can now ask the following natural question: Can a conventional string theory relax to a zero-energy supersymmetric vacuum while accelerating? In general this would require an involved analysis, and cannot be easily answered. However, there is a class of supersymmetric theories that can be readily investigated.

Let us suppose that the cosmological evolution of a theory is dominated by a single modulus field, which has a stable supersymmetric vacuum. The potential for such a field must then satisfy the stability form $V(\phi) = 2(D-2)((D-2)(\partial_\phi W)^2 - -\kappa^2 (D-1) W^2)$ and the superpotential W must have a critical point where $\partial_\phi W = 0$ which is the supersmymmetric vacuum of the theory [13, 14]. Many supergravity theories have potentials which satisfy this form. Here D is the dimension of spacetime.

In $4D$, this condition restricts the admissible potentials to the form

$$V = 8(\partial_\phi W)^2 - 12\kappa^2 W^2\,, \tag{4.1}$$

with $\kappa^2 = 8\pi/M_4^2$. If this potential is to have a zero-energy supersymmetric vacuum and to support $w = const$ quintessence-like evolution, then it must asymptotically have the form $V \sim \exp(-c\kappa\phi)$, where c must obey $|c| < \sqrt{2}$. We now show that this is inconsistent with (4.1).

If the theory is to give rise to an eternal $w = const$ quintessence universe while flowing towards the vacuum, since it needs an asymptotically exponential potential, with our notations the vacuum must be parameterized by $\phi \to \infty$. The superpotential which leads to such a form and *has* $\partial_\phi W \to 0$ as $\phi \to \infty$ can be expanded in a series of exponentials, with the leading term say

$$W = W_0 \exp(-\alpha\kappa\phi)\,, \tag{4.2}$$

where α is just a number. This gives rise to a potential

$$V = 8W_0^2\kappa^2\left(\alpha^2 - \frac{3}{2}\right)\exp(-2\alpha\phi)\,, \tag{4.3}$$

and this has the required asymptotic form. However, quintessence behavior also requires the positivity of the potential $V > 0$, which gives $|\alpha| > \sqrt{3/2}$ and hence

$$|c| = 2|\alpha| > \sqrt{6}\,. \tag{4.4}$$

Therefore a theory which asymptotically flows towards a supersymmetric stable vacuum $\partial_\phi W = 0$ satisfying (4.1) cannot do so while accelerating!

Note that we could have asked the question differently. We could view (4.1) as a differential equation defining W given a form of $V(\phi)$, and choose the form of V such that the cosmological evolution results in accelerated expansion. Such superpotentials exist, they do not have any critical points $\partial_\phi W = 0$ in the quintessence-like regime $\phi \to \infty$. Instead, a straightforward calculation shows that in this regime W and its derivative diverge. Hence an accelerating theory is not going towards a supersymmetric vacuum where $\partial_\phi W = 0$.

We stress that our analysis does not exclude inflation of the universe as a transient phenomenon. The universe could accelerate for a time and then switch to a stage of decelerated expansion, relaxing to a ground state. In this case the future infinity does not have spacelike portions, and there are no future horizons, just like the future in figure 1. Therefore this would not be in conflict with the current lore of string theory.

JHEP06(2001)003

5. Conclusion

The question raised in this paper concerns the possibility of defining in a precise way a set of observable quantities analogous to an S-matrix in an accelerating universe. Without such well defined quantities string theory as we know it would be impossible to formulate.

In deSitter space one might think of defining boundary correlators, analogous to the AdS observables. In this case the boundaries would be the space-like infinitely inflated past and future. Such correlators could be thought of as a kind of S-matrix relating the asymptotic contracting initial state to the asymptotically expanding final state. However it is clear that due to the existence of future horizons these are not observable by any real observer in the deSitter space. Perhaps such objects can be useful even if they are not observable but experience with black holes has taught us to be wary of such constructions. The alternative description of deSitter space in terms of a single causal diamond is certainly not suitable for an S-matrix description. The physics is that of a finite cavity with hot walls in which particles can get absorbed.

The main point of this paper is that the spaces produced by equations of state with $-1 < w < -1/3$ have future horizons and may be no better suited to an S-matrix or S-vector description. If this is so and observation continues to support an accelerated universe then we expect that the current set of concepts available in string theory will not be sufficient to give a coherent description of our universe.

Note added: as we were completing this paper, W. Fischler, A. Kashani-Poor, R. McNees and S.Paban informed of us their work [15] reaching similar conclusions.

Acknowledgments

It is a pleasure to thank Tom Banks for useful discussions. N.K. thanks the Institute for Theoretical Physics at Santa Barbara for its hospitality. This work was supported in part by NSF grants PHY-99-07949 and PHY-9870115. The work of S.H. is also supported by the D.O.E. under contract DE-AC03-76SF00098 and by a D.O.E. OJI grant.

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